As If Mathematics Were True

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An ongoing, and seemingly unending, philosophical debate is the *realism* versus *anti-realism* debate. On the one side are realists, claiming that objects exist in some realm that is independent of us and that it is in virtue of reference to these objects that our statements about them are true. Those making such assertions about the metaphysical realm we call metaphysical realists or platonists. Those making such assertions about the physical realm we call physical or scientific realists. On the other side are anti-realists, claiming that such positions are (typically epistemologically) untenable and so talk of such objects should be interpreted ideally, nominalistically, or fictionally.

In the first presentation on Plato (May 3rd), I argue for a version of *mathematical realism* that cuts a midpoint between these two philosophical poles. I first show that Plato himself keeps a clear distinction between mathematical and metaphysical realism and the knife he uses to slice the difference is method. The philosopher's *dialectical method* requires that we tether the truth of hypotheses to the existence of metaphysical objects. The mathematician's *hypothetical method*, by contrast, takes hypotheses *as if* they were first principles and so no metaphysical account of their truth is needed. Thus, we come to Plato's *methodological as-if realism*: in mathematics, we treat our hypotheses *as if* they were first principles, and, *consequently*, our objects *as if* they existed, and we do this with the purpose of solving mathematical problems.

In the second presentation (May 5th), I turn next to develop my own methodological as-if realism by comparing it to other structuralist views; I show that while these latter push us back to the same realist versus anti-realist debates, my *structural as-ifist* approach yet survives. Taking the road suggested by Plato, I argue that: some of our methodological commitments to taking our axioms as if they were first principles, will be made in light of *mathematical practice* (with the goal of solving mathematical problems); some will be made in light of *mathematical applicability* (with the goal of solving physical problems); and some will be made in light of *logical/philosophical considerations* (with the goal of solving meta-mathematical problems). Yet, none of these commitments will be made with the goal of solving metaphysical problems. Finally, I argue that it is in light of these later philosophical problems that we should take category theory *as if* it were a foundation for mathematical structuralism.

I conclude by considering where this leaves us with respect to the realism versus anti-realism debate, both in mathematics and in science. I claim that *mathematical realism* is to be properly understood as *as-if realism*: in mathematics there is nothing more to existence than what we can say. In science, by contrast, as-if realism is either idealism, nominalism or fictionalism, because in science there is more to existence than what we can say, viz., there is what we *must* show. What explains this difference is the following: mathematics is a language, it is not a science, it *talks about* objects without *being about* them; in contrast, physical science, as a science, must *be about* objects.